Title	JC and Polytechnic Mathematics Material Compilation –Pure Mathematics
Author	AprilDolphin
Date	12/11/2024

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Title	JC and Polytechnic Mathematics – Basic Operation of Complex Numbers
Author	?im ? ang Sheng, School of Information Technology, Nanyang Polytechnic ?CCA: ? ?? Mentoring Club?
Date	?/12/201?

This portion is designed to address a basics of complex numbers addition, subtraction, multiplication and division and does not involve any geometry.

Applicable to

C A evel H2 Mathematics

Nanyang Polytechnic, School of Engineering – Engineering Mathematics 1A

Nanyang Polytechnic, School of Information Technology – Diploma Plus Program (Mathematics)

Introducing the imaginary unit (Engineering students use instead)

$= \sqrt{-1}$

If you have read my document on secondary school algebra, you will realize that I specifically tell students not to use in any algebra operations (unless question ask you to do so, of course) as it is reserved for specific use. For a long time, you may be thinking what exactly it is being used for, it is in fact used to represent the value of $\sqrt{-1}$at higher level mathematics.

Basic Operation of Imaginary Numbers

Addition Rule

$+ \quad = (\quad + \quad)$

Subtraction Rule

$- \quad = (\quad - \quad)$

Multiplication Rule

$(\ ) = -1$

$(-\ )(\ ) = 1$

$(\quad) = (\ )(-1) = -$

Division Rule

$— = –$

Basic Examples:

Q1. $6 \quad + \; 8$

Q2. $5 \quad - \; 3$

Q3. $3 \; (4 \;)$

Q4. $\frac{2}{5}$

Q1	$6 \; + \; 8 \; = 1 \; 4$
Q2	$5 \; - \; 3 \; = \; 2$
Q3	$3 \; (4 \;) = \; -1 \; 2$
Q4	$\frac{2}{5} = \frac{2}{5}$

If a number contains both an imaginary and a real number, the number is called a complex number.

A complex number is in the following form

$$+$$

is the real part, is the imaginary part.

Complex Number Operations

Addition Rule

$$(\; + \;) + (\; + \;) = (\; + \;) + (\; + \;)$$

Subtraction Rule

$$(\; + \;) - (\; + \;) = (\; - \;) + (\; - \;)$$

Multiplication Rule (I hope you recall your secondary school expansion and factorization class, as it is similar to that, with one exception, that $(\;) = -1$)

$$(\; + \;)(\; + \;) = \; + \; + \; + (-1)$$

$$- \; + \; + \; = (\; - \;) + (\; + \;)$$

Division Rule (Students who took Additional Mathematics, this may ring a bell as it is similar to the method you used to rationalize denominator in secondary 3.)

Situation 1.

$$\frac{+}{+} = \frac{+}{+} \times \frac{-}{-}$$

Situation 2.

This method is called complex conjugate multiplication, a complex conjugate has the following properties that makes the above method work.

1. The conjugate multiplied to the complex number will always be equal to 1.
2. The sign between the imaginary and real part of the denominator has to flip to become the conjugate (i.e. "+" to "–" and "–" to "+").

Basic Examples
Q5.
Q6.
Q7.

Q8.

Q5.	By distributive law, the expression is equal to the following
Q6.	
Q7.	(Like how you expand expression brackets in secondary school, with an exception to note, being)

Q8.	The complex conjugate of the denominator of the expression —— is

Title	Complex Numbers Argument and Modulus [Converting from Algebraic Form to Polar and Exponential Form Form]
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [CCA: NYP Mentoring Club]
Date	15/12/2018

Applicable to

- Jun
- Nanyang Polytechnic, School of Engineering, Engineering Mathematics 1A
- Nanyang Polytechnic, School of Information Technology, Mathematics (Diploma Plus Program)

Basic Notation	
	The modulus of the complex number. (Also denoted in textbooks using the 2 vertical strokes. If is the complex number, the modulus can be written as | |.
	The argument of the complex numbers. (Denoted in textbooks as a r(g) given is the complex number.)

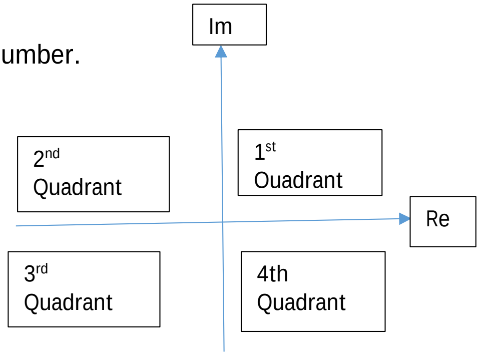
Notation Example	
+ (Algebraic Form)	Algebraic Interpretation: is the real part of the complex number and is the imaginary part of the complex number.
(Polar Coordinate Form)	Given complex number + refers to the distance of the point on the complex plane relative to the origin point of the complex plane. Can be computed using $= \sqrt{{}^{2} + {}^{2}}$ (Also known as the modulus) refers to angle of the line relative the positive real-axis, that start from the point of origin to the coordinate of + (Also known as the argument)
(c o(s) + s i(n)) (Polar Form)	Given complex number + refers to the distance of the point on the complex plane relative to the origin point of the complex plane. Can be computed using $= \sqrt{{}^{2} + {}^{2}}$ (Also known as the modulus)

	refers to angle of the line relative the positive real-axis, that start from the point of origin to the coordinate of + (Also known as the argument)
(Exponential Form)	Given complex number + refers to the distance of the point on the complex plane relative to the origin point of the complex plane. Can be computed using $= \sqrt{\ ^2 + \ ^2}$ (Also known as the modulus) refers to angle of the line relative the positive real-axis, that start from the point of origin to the coordinate of + (Also known as the argument)

As you can see the required values for polar coordinate, polar form and exponential form are computed using the same formula, so I will just need to guide you all on how to compute the modulus and argument.

Step 1. Sketch a complex plane and plot the coordinate of the complex number within the complex plane. (This is also called an argand diagram.)
Step 2. Draw a line, the line shall start from the origin point and reach the coordinate of the complex number.
Step 3. Determine the Quadrant Number of the Complex Number.

(Im means imaginary axis, Re means the real axis.)



Step 4. Compute Basic Angle of the Complex Number

Using $= \tan^{-1}(\frac{}{})$

Step 5. Use the following rules to deduce the argument
For first quadrant
$=$

For second quadrant
$= \; -$

For third quadrant

$$= - \quad +$$

For fourth quadrant

$$= -$$

Step 6. Compute using the below formula

$$= \sqrt{\ ^2 + \ ^2}$$

Find the modulus and argument of the following complex numbers.

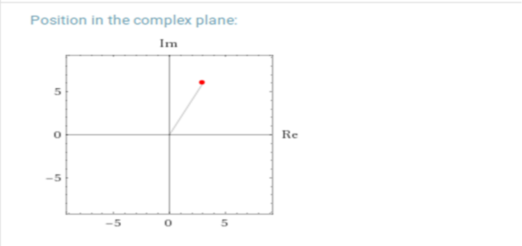
1. $3 + 6$
2. $1 - 3$
3. $-1 + 3$
4. $-1 -$

Question 1.

First quadrant therefore:

$$=$$

$3 + 6$ as mapped on the complex plane



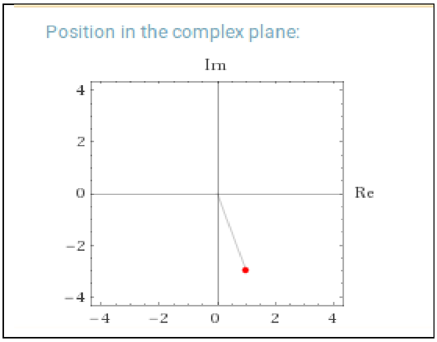
$$= \tan^{-1}\left(\frac{6}{3}\right) = 1\,1\;1$$

$$= \sqrt{6^2 + 3^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$$

Written in Polar Coordinate Form: $3\sqrt{5} \quad 1\,1\;1$	
Written in Polar Form $3\sqrt{5}\,(\cos(1\,1\;1) + \sin(1\,1\;1))$	
Written in Exponential Form $3\sqrt{5}^{\ (1\,1\;1)}$	

Question 2.

$1 - \ 3$ as mapped on the complex plane.



The complex position of the value lies in the fourth quadrant of the complex plane, therefore, $= \ -$

In this case, the opposite is the absolute value of the imaginary part while the adjacent is the real part of the complex number.

*When dealing with angles, always convert coordinate values to absolute values.

$$= t\ a\ n^{-1}\left(\frac{3}{1}\right) = 12\ 4\ 9$$

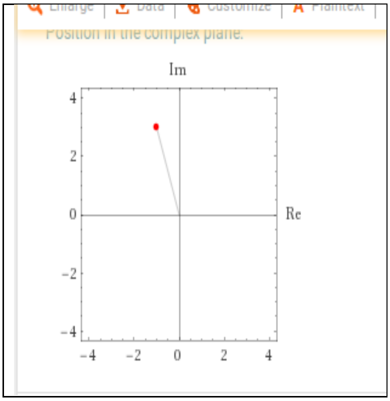
$$= -12\ 4\ 9$$

$$= \sqrt{1^2 + (-3)^2} = \sqrt{1\ 0}$$

Written in Polar Coordinate Form:	$\sqrt{1\ }0\ -\ 12\ 5$
Written in Polar Form	$\sqrt{1\ }0(c\ o\ s(-12\ 5) + \ s\ i\ n(-12\ 5))$
Written in Exponential Form	$\sqrt{1\ }0^{-12\ 5}$

Question 3.

$-1 + 3$ as mapped on the complex plane



Since in Second Quadrant, the Argument is computed as follows

$$= \quad -$$

$$= \tan^{-1}\left(\frac{3}{1}\right) = 12\ 4095$$

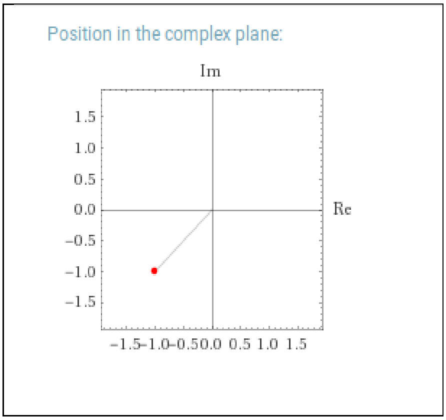
$$= \quad -124905 \qquad = 1.8\ 9$$

$$= \sqrt{3^2 + 1^2} = \sqrt{10}$$

Written in Polar Coordinate Form	$\sqrt{10}\ \ 1.8\ 9$
Written in Polar Form	$\sqrt{10}(\cos(1.8\ 9) + \ \sin(1.8\ 9))$
Written in Exponential Form	$\sqrt{10}(e^{1.8\ 9})$

Question 4.

$-1-$ as mapped on the complex plane.



Since the coordinate of the complex number lies in the third quadrant, we use the following formula

$$= - +$$

$$= \tan^{-1}\left(\frac{1}{1}\right) = 0.78540$$

$$= - + 0.78540 - 2.36$$

$$= \sqrt{1^2 + 1^2} = 1.4142$$

Written in Polar Coordinate Form	$\sqrt{2}\quad -2.36$
Written in Polar Form	$\sqrt{2}\ (s\ in(-2.36) + s\ in(-2.36))$
Written in Exponential Form	$\sqrt{2}\ (^{-2.36})$

Title	Differentiation of Inverse Trigonometric Functions
Author	AprilDolphin
Date	19/9/2024
Assumptions	This article assumes you already have good knowledge of using chain rule to differentiate functions as well as basic knowledge of finding derivatives of algebraic, logarithmic, exponential and trigonometric functions.

Function	Derivatives
$(\ ) = \sin^{-1}(\ )$	$'(\ ) = \frac{1}{\sqrt{1 - \ ^{2}}}$
$(\ ) = \cos^{-1}(\ )$	$'(\ ) = \frac{-1}{\sqrt{1 - \ ^{2}}}$
$(\ ) = \tan^{-1}(\ )$	$'(\ ) = \frac{1}{1 + \ ^{2}}$

Chain Rule in General
Given $(\ ) = \ (\ (\ ))$ $'(\ ) = \ '(\ )\ '(\ )$

Application of Chain Rule to Inverse Trigonometric Functions	
Function	Derivatives
$(\ ) = \sin^{-1}(\ )$	$'(\ ) = \frac{1}{\sqrt{1 - \ ^{2}}}(\ ')$
$(\ ) = \cos^{-1}(\ )$	$'(\ ) = \frac{-1}{\sqrt{1 - \ ^{2}}}(\ ')$
$(\ ) = \tan^{-1}(\ )$	$'(\ ) = \frac{1}{1 + \ ^{2}}(\ ')$

Example 1.

Find the derivative of the following functions

(a) $y = \sin^{-1}(\ ^{6}\ )$

Using chain rule $\frac{}{} = \frac{}{} \times \frac{}{}$

$$\frac{}{} = \frac{1}{\sqrt{(1-\ ^{2})}} \times 6\ ^{6} = \frac{6\ ^{6}}{\sqrt{1-\ ^{12}}}$$

(b) $= \tan^{-1}(2\ )$

Using chain rule $\frac{}{} = \frac{}{} \times \frac{}{}$

$$\frac{1}{1+(2\ )^{2}} \times 2\ \ln(2) = \frac{2\ \ln(2)}{1+2^{2}}$$

(c) $(\ ) = \cos^{-1}(\frac{}{3})$

Using chain rule, where any composite function $(\ ) = (\ (\ ))$ will produce a derivative of $'(\ )\ '(\ )$

$$\cos^{-1}(\frac{}{3}) = \cos^{-1}(\frac{1}{3}\ )$$

$$'(\ ) = \frac{-1}{\sqrt{1-(\frac{1}{3}\ )^{2}}} \times \frac{1}{3} = -\frac{1}{3\sqrt{1-\frac{1}{9}\ ^{2}}} = -\frac{1}{\sqrt{9}\sqrt{1-\frac{\ ^{2}}{9}}}$$

$$= -\frac{1}{\sqrt{9-\frac{\sqrt{9}^{2}\ ^{2}}{9}}} = -\frac{1}{\sqrt{9-\ ^{2}}}$$

(d) $= \ ^{\sin^{-1}(2\ )}$

Using chain rule, where $\frac{}{} = \frac{}{} \times \frac{}{}$

$$2^{\sin^{-1}(2\ )} \times \frac{1}{\sqrt{1-(2\ ^{2})}} = \frac{2^{\sin^{-1}(2\ )}}{\sqrt{1-4\ ^{2}}}$$

Title	Differentiation of Implicit Functions
Author	AprilDolphin
Date	27/9/2024

Definition of an explicit function, is where can be defined as exactly equal to $(\ )$or written mathematically, $= \ (\ )$

Definition of an implicit function is where cannot be written as equal to solely just $(\ )$but requiring the person to write or rewrite a function as where $(\ ,\ ) = 0$

Examples of Explicit Functions	Examples of Implicit Functions
$= {}^{2} + \mathrm{l}(n) + 2 \ + 4$	${}^{2} + {}^{2} + 2 \ = 25$
$= \mathrm{s}\ \mathrm{i}(n^{4}) + {}^{5} - \mathrm{t}\ \mathrm{a}(n)$	${}^{3} - \frac{5}{} = 6\,{}^{2}\,{}^{6} + 9 \ - 64$
$= {}^{\mathrm{s\ i}(n)} - \mathrm{c}\ \mathrm{o}(\mathrm{s}) + 5$	${}^{5} + \mathrm{s}\ \mathrm{i}(n) = {}^{2} + {}^{5}$

Rules of Differentiation Necessary to Ensure a Smooth Understanding of Implicit Differentiation:

Chain Rule: $(\ ) = (\ (\ )),\ {}'(\ ) = {}'(\ )\ {}'(\ )$

Product Rule: $(\ ) = (\ )\ (\ ),\ {}'(\ ) = {}'(\ )\ (\ ) + {}'(\ )\ (\ )$

Quotient Rule: $(\ ) = \frac{(\ )}{(\ )},\ {}'(\ ) = \frac{(\ )\ {}'(\ ) - (\ )\ (\ )}{[\ (\ )]^{2}}$

Instruction to Implicit Differentiation

1. Differentiate left side and right side of implicit function with respect to
2. Identify term where is differentiated, multiplying the derivative of $(\ )$by —
3. Group terms with — together on one side and without on the other side
4. Take out common factor of — on the side with the term of —
5. Remove terms without the factor of — by division, multiplication or any applicable method

Example 1.

Find the derivative of the following implicit function

$\;^{2}\;\;^{2}+\quad=\quad-1\;0$

Step 1. Differentiate left hand side and right-hand side of implicit function with respect to .

$$\frac{}{}\;[\;^{2}\;\;^{2}+\quad]=\frac{}{}\;[\;-1\;0]$$

Step 2. Differentiating term by term, we get the following and can identify the term where is differentiated then multiply the derivative by $\frac{}{}$

Terms	Derivatives
$\;^{2}\;\;^{2}$ using product rule →	$[2\;(\;^{2})]\times\frac{}{}+(2\;)(\;^{2})$
using power rule →	1
	$1\times\frac{}{}$
$-1\;0$	0

Putting all derivative together, we would get the following value:

$$[2\;(\;^{2})]\times\frac{}{}+(2\;)(\;^{2})+1=1\times\frac{}{}$$

Step 3. Group terms with $\frac{}{}$ together on one side and without on the other side

$$2\;\;^{2}\left(\frac{}{}\right)+2\;\;^{2}+1=\frac{}{}$$

$$2\;\;^{2}\left(\frac{}{}\right)-\frac{}{}=-2\;\;^{2}-1$$

Step 4. Take out common factor of $\frac{}{}$ on the side with the term of $\frac{}{}$

$$(2\;\;^{2}-1)\left(\frac{}{}\right)=-2\;\;^{2}-1$$

Step 5. Remove terms without the factor of — by division, multiplication or any available method

$$\text{—} = \frac{-2 \quad ^{2} - 1}{2 \quad ^{2} - 1}$$

Example 2. Differentiate the following implicit function

$$^{2} = 2\ ^{3} + \quad + 7$$

$$\text{—}\ [\ ^{2}] = \text{—}\ [2\ ^{3} + \quad + 7]$$

$$2 \quad (\text{—}) = 6\ ^{2} + 1\ (\text{—}) + 0$$

$$2 \quad (\text{—}) - 1\ (\text{—}) = 6\ ^{2}$$

$$\text{—}\ (2 \quad - 1) = 6\ ^{2}$$

$$\text{—} = \frac{6\ ^{2}}{2 \quad - 1}$$

Example 3.

$$2 \quad + \ ^{2} - 3 \quad - \ ^{2}\ ^{5} = 0$$

$$\text{—}\ [2 \quad + \ ^{2} - 3 \quad - \ ^{2}\ ^{5}] = \text{—}\ [0]$$

Terms	Derivative
2	$2\ (1) \times \text{—} + 2$
^{2}	$2 \quad \times \text{—}$
-3	-3
$-\ ^{2}\ ^{5}$	$-2 \quad ^{5} + 5\ ^{4}(- \ ^{2}) \times \text{—}$

0	0

$$2\ (\text{—})+2\ \ +2\ (\text{—})-3-2\ \ ^{5}+5\ ^{4}(-\ ^{2})\times\text{—}\ =0$$

$$2\ (\text{—})+2\ (\text{—})+2\ \ -3-2\ \ ^{5}-5\ ^{2}\ ^{4}(\text{—})=0$$

$$2\ (\text{—})+2\ (\text{—})-5\ ^{2}\ ^{4}(\text{—})=-2\ \ +3+2\ \ ^{5}$$

$$\text{—}\ (2\ \ +2\ \ -5\ ^{2}\ ^{4})=-2\ \ +3+2\ \ ^{5}$$

$$\text{—}\ =\frac{-2\ \ +3+2\ \ ^{5}}{2\ \ +2\ \ -5\ ^{2}\ ^{4}}$$

Example 4.

$$+\ \ ^{2}+\ \ \ =5-\sin(^{3}\ \ )$$

$$\text{—}\ [\ \ +\ \ ^{2}+\ \ \ ]=\text{—}\ [5-\sin(^{3}\ \ )]$$

$$1(\text{—})+2\ (\text{—})+\ \ \ +\ \ (\text{—})=0-3\ ^{3}\ \cos^{3}\ (\text{—})$$

$$1(\text{—})+2\ (\text{—})+\ \ (\text{—})+3\ ^{3}\ \cos(^{3}\ )(\text{—})=-$$

$$\text{—}\ [1+2\ \ +\ \ \ +3\ ^{3}\ \cos(^{3}\ )]=-$$

$$\text{—}\ =-\frac{}{1+2\ \ +\ \ \ +3\ ^{3}\ \cos(^{3}\ )}$$

Title	Parametric Differentiation
Author	AprilDolphin
Date	11/11/2024

Definition of Parametric Functions

A parametric function is a function that express relationships of 2 variables using a third variable, in which the third variable is called a parameter.

Example of a Parametric Function as follows

$= {}^2 + 5 \ - 4$

$= 2 \ + 6$

In the above case, the relationship between and is expressed using parameter .

Simplest examples of applications of parametric functions:

Kinematics Modeling where a flying object (such as a bird or fighter jet) trajectory

follows a curve that goes up and down repeatedly.

In this above case, it would be extremely difficult to express the variables in the non-parametric way and using a parametric function for kinematic modelling is appropriate in this case in the following manner.

The forward motion (being in acceleration) over time can be written as

$(\ ) = 0.8 \ + 3 \ {}^2$

The altitude of the flying object in up and down motion over time can be written as

$(\ ) = 5 + c\ o(s)$

Differentiation involving parametric functions can be done using a technique derived from chain rule.

Let be the first variable, be the second variable and be the parameter involved and our goal is to find the change in with respect to .

$$\frac{}{} = \frac{(\frac{}{})}{(\frac{}{})} = \frac{}{} \times \frac{}{}$$

1. Find — given that $= 4\ ^{2} + 3\ - 1$and $= 3\ ^{4} - 2$

— $= 8\ + 3$

— $- 1\ 2^{3} - 2$

$$— = \frac{(—\ )}{(—\ )} = \frac{(8\ + 3)}{(1\ 2^{3} - 2)}$$

2. Find — for the parametric equation $= 2\,s\ i(\ n)$and $= 3\,c\ o(\ s)$

— $= -3\,s\ i(\ n)$

— $= 2\,c\ o(\ s)$

$$— = \frac{(—\ )}{(—\ )} = \frac{-3\,s\ i(\ n)}{2\,c\ o(\ s)} = -\frac{3}{2}\,t\ a(\ n)$$

3. A curve is defined parametrically by $= \frac{2}{+1}$ and $= \frac{2}{+1}$.
Find —.

Using parametric differentiation, we can find the derivatives of — and — as follows.

Quotient rule where — $\left[\frac{(\ )}{(\ )}\right] = \frac{(\ )\ '(\ ) - (\ )\ '(\ )}{[\ (\ )]^2}$

$$\text{—} = \frac{(\ +1)(2\ ) - 1(\ ^2)}{(\ +1)^2} = \frac{2\ ^2 + 2\ - \ ^2}{(\ +1)^2} = \frac{\ ^2 + 2}{(\ +1)^2}$$

$$\text{—} = \frac{(\ +1)(2) - 2\ (1)}{(\ +1)^2} = \frac{2\ + 2 - 2}{(\ +1)^2} = \frac{2}{(\ +1)^2}$$

$$\text{—} = \text{—} \times \text{—} = \frac{\ ^2 + 2}{(\ +1)^2} \times \frac{(\ +1)^2}{2} = \frac{\ ^2 + 2}{2}$$

Title	Integration by Substitution
Author	AprilDolphin
Date	2/10/2024

Basic Introduction to Integration by Substitution

Integration by Substitution corresponds to chain rule as taught in differentiation and as a result it is sometimes also called the reverse chain rule as well.

Important Concepts to Note as I would be using the concept quite frequently here:

— —

Instructions to Integration by Substitution.

Step 1. Examine the questions carefully and define the value of as mentioned in the question.

Step 2. Differentiate with respect to .

Step 3. Express in terms of

Step 4. Rewrite the original integral in terms of and

Step 5. Integrate according to relevant formula

Step 6. Replace with the original expression of in step 1.

Example 1.
Evaluate

Step 1. Set

Step 2. Differentiate with respect to :

Step 3. Express in terms of : — , hence, — —

Step 4 Rewrite the original integral in terms of and : —

—

Step 5: Integrate according to relevant formula: – – —— – — —

Step 6: Replace with the original expression of in step 1: — ———

Question 1.
Evaluate

Set

Hence, —

— —

— —

——— — ————— —————

Question 2.
Evaluate

Set

— , therefore — –

–

– — ——————

Question 3.

Evaluate using the substitution

Set

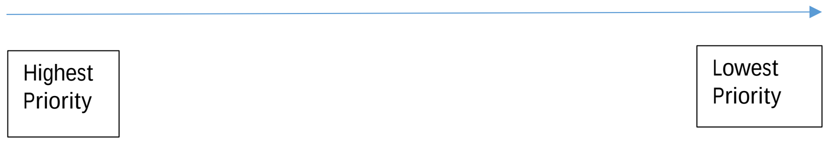
Title	Integration by Parts
Author	AprilDolphin
Date	26/10/2024

Formula for Integration by Parts

$$\int \quad = \quad - \int$$

In any integration by parts application, we choose the value of using a rule where the type of function mentioned first has the highest priority and the one mentioned second has lower priority and so on, the one mentioned the last will have the lowest priority.

The rule is called



In short, the rule is called the L.I.A.T.E rule.
The purpose of complying to the above rule is to ensure the function we selected is made easy by integration by parts rather than being made even more complicated. As seen with the below example.

Example 1.

$$\int$$

If we select $=$, in compliance with the rule mentioned above, we literally get the following:

$$= \quad \text{and} \quad =$$

$$= \int \quad =$$

$$\int \quad =$$

$$= 1$$

As a result, we combined them together using the integration by parts formula

$\int \quad = \quad - \int$

$- \quad (1) = \quad - \quad +$

However, if we reversed the roles of and above, we get the following

$= \quad , \quad =$

$= \quad , \quad = \frac{2}{2}$

Combining them together, using the integration by parts formula, we get the following integral, literally,

$(\frac{2}{2}) - \int \frac{2}{2}$

Which becomes as difficult or even harder to integrate compared to the function we

Question 1.
Find $\int \mathrm{l\ n}$

Which can be rewritten as

$\int \mathrm{l\ n}\ (1)$

Using the L.I.A.T.E rule explained earlier, we can choose $= \mathrm{l}$ nand therefore, $= 1$

$= \mathrm{l\ n}$
$=$

$\int \frac{1}{} \quad = \int 1 \quad =$

Combining them, we get $\mathrm{l}\ (\mathrm{n}\ ) - \quad +$
Question 2.
Find $\int {}^{2}\ \mathrm{l\ n}$

Selecting $= \ln$ and therefore $= {}^2$

$$= \int {}^2 = \frac{{}^3}{3}$$

$$\int \quad = \int \frac{{}^3}{3}\left(\frac{1}{}\right) = \int \frac{1}{3}{}^2 = \frac{\left(\frac{1}{3}\right)}{2+1}{}^{2+1} = \frac{1}{9}{}^3$$

$$\ln(\ )\left(\frac{{}^3}{3}\right) - \frac{{}^3}{9} +$$

Question 3.
Find $\int \cos$

Selecting $=$ and therefore $= \cos$

$$= \int \cos = \sin$$

$$\int \quad = \int \sin()\,1 \quad = -\cos$$

$$\sin + \cos +$$

Question 4.
Find $\int \tan^{-1}$

Select $= \tan^{-1}$ and $= 1$

$$=$$

$$\int \quad = \int \quad \left(\frac{1}{{}^2+1}\right) = \int \frac{}{{}^2+1} = 0.5\ln({}^2+1)$$

$$\tan^{-1}(\ ) - 0.5\ln({}^2+1)$$

Title	Differential Equations –Basic Concepts and First Order Differential Equations (Direct Integration and Separation of Variables)
Author	AprilDolphin
Date	30/10/2024

Differential Equations are equations that contain derivatives, where the so-called
“Solutions” are the function(s) that can s
indefinite integral of derivatives is generally considered solving simple differential equations.

Concepts to understand in any differential equations

- Order –the order of a differential equation is simply the order of the highest derivative in the differential equation.
- Degree –the degree of a differential equation is simply the power of the highest derivative of the differential equation.

Example

1. Given the following three examples, state the order and degree of the differential equation:

   (a) —

   (b)

   (c) — —

1(a) Highest derivative is the third derivative –third order differential equation
Highest power of the highest derivative is 1 –first degree differential equation
Third order, first degree differential equation.

1(b) Highest derivative is the second derivative –second order differential equation
Highest power of the highest derivative is 2 –second degree differential equation
Second order, second degree differential equation

1(c) Highest derivative is the second derivative –second order differential equation
Highest power of the highest derivative is 3 –third degree differential equation
Second order, third degree differential equation.

After understanding the above, we can go on and understand the concept of independent and dependent variables.

Independent variables in any differential equation are the variables with respect to where differentiation occur and dependent variables are the variables being differentiated.

In the below example, identify the dependent and independent variables.
Example
2.

——

is the dependent variable

is the independent variable

Q1. Solve the following differential equation using direct integration

(a) —

Move to the other side by multiplying both sides by to get the following

Integrate both sides to get

Using integration by parts for the right-hand side, we get the following

Set ,

–

–

Q2. Solve the following differential equation by separation of variables.

Multiply both sides by

Integrate both sides to get the following

Q3. Solve the following differential equation by separation of variables.

Cross-multiply both sides denominator and we get the following

Cross multiply again to get

Divide both sides by 3